562

VOLUME 80 SEPARATE No. 562

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

DECEMBER, 1954

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

MOMENTS AND DEFLECTIONS IN STEEL GRID BRIDGE FLOORS

by H. J. Greenberg

STRUCTURAL DIVISION

(Discussion open until April 1, 1955)

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

THIS PAPER

--represents an effort by the Society to deliver technical data direct from the author to the reader with the greatest possible speed. To this end, it has had none of the usual editing required in more formal publication procedures.

Readers are invited to submit discussion applying to current papers. For this paper the final date on which a discussion should reach the Manager of Technical Publications appears on the front cover.

Those who are planning papers or discussions for "Proceedings" will expedite Division and Committee action measurably by first studying "Publication Procedure for Technical Papers" (Proceedings — Separate No. 290). For free copies of this Separate—describing style, content, and format—address the Manager, Technical Publications, ASCE.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

MOMENTS AND DEFLECTIONS IN STEEL GRID BRIDGE FLOORS

H. J. Greenberg[1]

SYNOPSIS

Formulas are proposed for the calculation of the maximum deflection and maximum negative edge-moment occuring in a steel-grid highway bridge floor under wheel loads. Separate formulas are given for concrete-filled steel grid floors and open-grid steel floors respectively. These formulas take into account the basic anisotropy of such floors as a variable and are designed to fit the data tabulated by K. C. Kuo who carried through a complete analysis of the problem.[2] For completeness, the results of Kuo's analysis are summarized including the formulas he proposed for the calculation of maximum positive bending moments in such floors. Graphs of the deflection and moment coefficients over the required ranges of span and the anisotropy variable are included.

It is proposed that the formulas presented in this paper be used as a basis for specifications and design of steel-grid highway bridge floors.

INTRODUCTION

In a recent paper,[2] K. C. Kuo gave a complete solution of the problem of determining moments and deflections in both simple and one-way continuous, anisotropic, rectangular slabs under central loads. Kuo applied this general theory to the problem of determining the moments and deflections in steel grid highway bridge floors under wheel loads. This provided the first satisfactory analysis of steel grid floors taking into account the basic anisotropy of such floors and thus extended the work of H. M. Westergaard,[3] who considered the case of long, isotropic, simple slabs under concentrated loads. Due to the restriction to isotropy, Westergaard's theory, though applicable to the case of reinforced concrete slabs, does not apply to the case of steel grid slabs which vary in degree of anisotropy over a wide range. Nevertheless, in the absence of a more complete analysis, Westergaard's formulas were used as a basis of design for steel grid floors.[4] On the basis of his extended analysis Kuo

---

1. Assoc. Prof. of Math., Carnegie Inst. of Tech., Pittsburgh, Pa.
2. K. C. Kuo, "Moments in Steel Grid Highway Bridge Floors," Proceedings, Convention Group Meetings, AASHO, Omaha, 1951. This paper was presented by the present author who also prepared the final published draft of the paper when Dr. Kuo returned to China. All of Dr. Kuo's original computations have been available to the author in the preparation of the present paper.
3. H. M. Westergaard, "Computation of Stresses in Bridge Slabs Due to Wheel Loads," Public Roads, Vol. II, No. 1, March 1930.
4. AASHO Standard Specifications for Highway Bridges, 1949. See footnotes 7 and 8.

proposed new formulas to be used for design. These formulas give the maximum positive bending moments under a standard loading as functions both of span and a basic anisotropy parameter. The formulas are given for two separate ranges of the anisotropy parameter; one corresponding to concrete-filled, steel grid floors, the other to open-grid floors.

For purposes of design, in addition to the maximum positive bending moment, information is required as to the maximum deflection and the maximum negative moment occuring over a support (which may under some conditions govern the design). While Kuo tabulated the deflections and the negative edge-moments, he did not propose practical formulas to fit this data which could be used for specifications in design (it should be explained that the theoretically obtained formulas from which Kuo originally tabulated the deflections and moments were in the form of complicated infinite and double infinite series unsuited for use as design formulas). It is the primary purpose of the present paper to supplement Kuo's paper by providing convenient formulas for the deflections and negative edge moments. These formulas accurately fit Kuo's tabulated data as functions of span and the anisotropy parameter. Separate formulas are provided for the ranges of the anisotropy parameter corresponding to concrete-filled steel grid floors and open grid floors respectively. The secondary purpose of the present paper is to bring the results of Kuo's analysis to the attention of a wider audience and by collecting Kuo's formulas with the ones being proposed at present provide a convenient reference of basic formulas for use in the design of steel grid highway bridge floors. Graphs of the moment and deflection coefficient curves are also included. In the next section we review the results of Kuo's analysis.

Analysis of Steel Grid Floors

Steel grid floors may be treated as homogeneous, anisotropic, elastic, rectangular slabs with principal directions parallel to the edges. The stringers supporting the floor divide it into rectangular panels forming a one-way, continuous slab simply supported on rigid supports. Each panel may carry a load and the problem is to determine the bending moments and deflection of the continuous slab under the given loads.

Consider one panel of the continuous slab of span S and length b (Fig. 1), with flexural rigidities D_x, D_y and twisting rigidity H[5]. The deflection $w_c = w_c(x,y)$ at any point (x,y) in this panel can be obtained by superposition as the sum of the deflection $w = w(x,y)$ of the panel considered as a simple slab under the load distribution $q = q(x,y)$ which is actually being applied to this panel and the deflection $w' = w'(x,y)$ of the panel due solely to the edge moments $M_L(y)$ and $M_R(y)$ transmitted from the panels on the left and right of the given panel. Thus,

$$w_c = w + w' \tag{1}$$

The deflection $w(x,y)$ of the panel treated as a simple slab under the applied loads $q(x,y)$ must satisfy the partial differential equation[5]

5. The elastic constants D_x, D_y and H are defined in S. Timoshenko, "Theory of Plates and Shells," McGraw-Hill, 1940, Chap. V. There, too, can be found a derivation of the basic equations governing anisotropic slabs. Note that for a grid, which is not homogeneous, quantities such as D_x, D_y, H are computed as blockwise averages.

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q \qquad (2)$$

subject to the boundary conditions of zero deflection and moment. Following the Navier method,[5] used also in the isotropic case by Westergaard,[3] the solution w is obtained as a double infinite series of trigonometric functions of the two variables x and y chosen so as to satisfy the boundary conditions. The coefficients are obtained by expanding the given load function q in a similar series and then substituting these series into (2) and solving for the coefficients of w by equating coefficients of corresponding terms.

The deflection $w'(x,y)$ of the panel due solely to the transmitted edge moments $M_L(y)$, $M_R(y)$ is obtained by Kuo as follows. A trigonometric series in y is assumed for each of the two functions $M_L(y)$ and $M_R(y)$ and indeed for the moment distribution along any intermediate support separating two panels of the continuous slab. The moments along the simply supported edges of the two terminal slabs are of course zero. Assuming the deflection w of each treated as a simple slab to have been found as above, and the deflection w' of each panel due to its edge moments to be expressed in terms of these moments, then the condition of continuity of the slope of the slab across an intermediate support leads to an equation involving the edge-moments over three successive supports. This is a generalized three-moment equation for the one-way continuous slab and from this equation the coefficients of the expansions of the edge-moment functions are determined. This in turn determines the deflection w' and the total deflection w_c is determined from (1).

Kuo carries the above through in general terms and so finds the deflection for an arbitrary panel in a one-way continuous, anisotropic slab under arbitrary loading. He then applies these results to determining the maximum moments and deflections in steel grid bridge floors under wheel loads as follows.[6]

The wheel load on a panel is represented by a uniform load over a square area located in the center of a panel. This, except in extreme cases, leads to maximum positive moments at the center of the panel. Assuming all panels of the continuous slab to be identical, the maximum positive bending moment occurs when the number of panels is two and one of the panels is centrally loaded as in Figure 2. The uniform central load is given by $P = q_0 (kS)^2$ where q_0 is the uniform load per unit area and kS is the length of a side of the loaded square area expressed as a fraction k of the span S. Under this loading, the deflection w of the unloaded panel treated as a simple slab is zero. The deflection w of the loaded panel treated as a simple slab is found by the Navier method described above to be given by the double infinite series

$$w = \frac{16 q_0 a^4}{\pi^6 D_x} \sum_{m,n=1,3,\dots}^{\infty} A_{mn} \cdot \cos\frac{m\pi x}{S} \cdot \cos\frac{n\pi y}{b} \qquad (3)$$

---

6. We have, for convenience, in places made changes in Kuo's notations. These are rather obvious and require no special mention.

where

$$A_{mn} = \frac{\sin\frac{mk\pi}{2} \cdot \sin\frac{nSk\pi}{2b}}{mn\left[m^4 + 2m^2n^2\left(\frac{S}{b}\right)^2 s + n^4\left(\frac{S}{b}\right)^4 r\right]}$$

and where the summation is a double summation extended over odd integral values of the indices m and n and the letters r and s stand for the rigidity ratios

$$r = \frac{D_y}{D_x}, \qquad s = \frac{H}{D_x} \tag{4}$$

which are measures of the anisotropy of the slab ($r = s = 1$ corresponds to the case of isotropy). We now define the dimensionless deflection coefficient w^* corresponding to w , which when evaluated at the center of the loaded panel is given by

$$w^* = \left.\frac{w D_x}{PS^2}\right|_{(0,0)} = \frac{16}{\pi^6 k^2} \sum_{m,n=1,3,\cdots}^{\infty} A_{mn} \tag{5}$$

The bending moments at the center of the loaded panel corresponding to the deflection w are given in terms of the second partial derivatives of w by the formulas[5]

$$\left.\begin{aligned} M_x &= -\left.\left(D_x \cdot \frac{\partial^2 w}{\partial x^2} + D_1 \cdot \frac{\partial^2 w}{\partial y^2}\right)\right|_{(0,0)} \\ M_y &= -\left.\left(D_y \cdot \frac{\partial^2 w}{\partial y^2} + D_1 \cdot \frac{\partial^2 w}{\partial x^2}\right)\right|_{(0,0)} \end{aligned}\right\} \tag{6}$$

where D_1 is an elastic constant.[5]

The continuity condition and the resulting three moment equation determines the coefficients in the series for the interior edge-moments

$$M_e = \sum_{n=1}^{\infty} E_n \cdot \cos\frac{n\pi y}{b} \tag{7}$$

to be given by

$$\frac{E_n}{P} = \frac{16\,\rho\theta_1}{\pi^5 k^2} \left(\frac{\cosh 2n\theta_1 - \cos 2n\theta_2}{\sin 2n\theta_2 - \rho \sinh 2n\theta_1}\right) \cdot T(k,n) \tag{8}$$

where

$$T(k,n) = \sum_{m=1,3,\cdots}^{\infty} (-1)^{\frac{m-1}{2}} \cdot mn\, A_{mn} \quad , \tag{9}$$

and

$$\left.\begin{aligned} \theta_1 = \frac{S\pi}{b}\sqrt{\tfrac{1}{2}(\sqrt{r}+s)} \quad , \quad \theta_2 = \frac{S\pi}{b}\sqrt{\tfrac{1}{2}(\sqrt{r}-s)} \\ \rho = \frac{\theta_2}{\theta_1} \end{aligned}\right\} \tag{10}$$

The contribution of continuity to the deflection of the loaded panel is then shown by Kuo to be

$$w' = \frac{8PS^2}{\pi^5 k^2 D_x} \sum_{n=1,3,\cdots}^{\infty} \left[A_n(x) - B_n(x)\right] \cdot T(k,n) \cdot \cos\frac{n\pi y}{b} \tag{11}$$

where

$$A_n(x) = \frac{1}{C_n} \cdot \sinh\, n\left(\xi x - \frac{\theta_1}{2}\right) \cdot \sin\, n\left(\eta x + \frac{3\theta_2}{2}\right)$$

$$B_n(x) = \frac{1}{C_n} \cdot \sinh\, n\left(\xi x + \frac{3\theta_1}{2}\right) \cdot \sin\, n\left(\eta x - \frac{\theta_2}{2}\right)$$

$$C_n = n^2\theta_1 \cdot (\sin 2n\theta_2 - \rho \sinh 2n\theta_1) \; ,$$

and where $\xi = \frac{\theta_1}{S}$, $\eta = \frac{\theta_2}{S}$. We now define the dimensionless deflection coefficient w'^{*} corresponding to w', which when evaluated at the center of the loaded panel is given by

$$w'^{*} = \left.\frac{w' D_x}{PS^2}\right|_{(0,0)} = -\frac{8}{\pi^5 k^2} \sum_{n=1,3,\cdots}^{\infty} \frac{1}{n} \cdot T(k,n) \cdot \phi(n\theta_1, \rho) \tag{12}$$

where

$$\phi(\theta,\rho) = \frac{\sinh\frac{3\theta}{2} \cdot \sin\frac{\rho\theta}{2} - \sinh\frac{\theta}{2} \cdot \sin\frac{3\rho\theta}{2}}{\theta \cdot (\rho \sinh 2\theta - \sin 2\rho\theta)} \tag{13}$$

The bending moments M_x' and M_y' at the center of the loaded panel due to the continuity contribution w' to the deflections are given in terms of the second partial derivatives of w' by formulas exactly analogous to (6), with w replaced by w' as given by (11) and M_x, M_y replaced by M_x', M_y'.

The final formulas for the total deflection and total bending moments are given by superposition to be

$$w_c = w + w' \tag{1}$$

$$\left.\begin{aligned} M_{cx} &= M_x + M_x' \\ M_{cy} &= M_y + M_y' \end{aligned}\right\} \tag{14}$$

In terms of dimensionless quantities, the <u>maximum deflection coefficient</u> is given by

$$w_c^* = w^* + w'^* \tag{15}$$

where w^* is given by (5) and w'^* by (12). The maximum positive bending moment coefficients are just

$$\left.\begin{aligned} &\frac{M_{cx}}{P} \\ &\frac{M_{cy}}{P} \end{aligned}\right\} \tag{16}$$

since the bending moments are actually bending moments per unit of length in the x or y directions.

The edge-moment M_e becomes a maximum when both panels are subject to the square centrally located load $P = q_0 (kS)^2$, as in Figure 3. This is twice the value for the case of one panel loaded and so can be directly obtained from (7). The <u>coefficient of edge-moment</u>, taken at the center of the edge to yield the maximum value, for the case of both panels loaded is given by

$$M_e^* = -\frac{32}{\pi^5 k^2} \sum_{n=1,3,\cdots}^{\infty} \frac{1}{n} \cdot T(k,n) \cdot \Theta(n\theta_1, \rho) \tag{17}$$

where

$$\Theta(n\theta_1, \rho) = \frac{n\theta_1 \cdot (\cosh 2n\theta_1 - \cos 2\rho n\theta_1)}{\sinh 2n\theta_1 - \frac{1}{\rho} \sin 2\rho n\theta_1} \tag{18}$$

The coefficients of deflection and moments were tabulated by Kuo for the

ranges $.005 \leq r < 1.$ and $0 \leq \frac{S}{\sqrt{r}} \leq 1.$ which cover the cases of interest for application to steel grid slabs. The values of the ratio $\frac{S}{b}$ used ranged between 1/3 and 1, it being expected that, as in the isotropic case where the bending moment for $\frac{S}{b} = \frac{1}{3}$ differs by only 1% from that for $\frac{S}{b} \rightarrow 0$, there would be little change over the range $0 \leq \frac{S}{b} \leq \frac{1}{3}$. The ratio k of the side of the central square load to the span was varied between .1 and 1. In order to have the span S appear explicitly rather than the dimensionless variable k , the loaded area representing the wheel load was assumed to be 15" square. Thus, $kS = 1.25'$ and the range of k from .1 to 1 corresponds to a range of span values from $1.25'$ to $12.5'$.

In general it was found that the variation of the coefficients with the anisotropy parameter $\frac{S}{\sqrt{r}}$ which enters through the quantities defined in (10) was small. To eliminate the explicit dependence on this variable in the final proposed formulas, the value $\frac{S}{\sqrt{r}} = 1$ was chosen as representative for concrete-filled steel grids while for the open grids the value $\frac{S}{\sqrt{r}} = .05$ was selected as average.

For the concrete filled steel grid floors the constant D_1 which enters in (6) is assumed to be approximately given by μD_x where μ is the Poisson ratio of the slab and is taken to be .15 for the computations. In Figure 4, the solid lines show the variation of the bending moment coefficient $\frac{M_{cx}}{P}$ in the x direction with span S for $r = .1, .2, .4, .7,$ and 1.0 which covers the range of r for concrete-filled grid floors. The broken line shows the graph of $\frac{M_{cx}}{P}$ vs. S from the formula given in the 1949 AASHO specifications.[7] Since this curve is for an isotropic slab it must be compared with the solid curve for $r = 1.0$. This furnishes a check on Kuo's analysis against Westergaard's and the agreement is seen to be good. The variation with anisotropy from the AASHO specified curve is clearly shown. In Figure 5, the moment coefficient $\frac{M_{cy}}{P}$ in the y direction is plotted against span for a range of values of r . The 1949 AASHO specifications do not refer to M_{cy}, however, Kuo points out that even though M_{cy} is in general smaller than M_{cx},, the slab is generally weaker in the y direction than in the x direction, i.e. D_y is smaller than D_x, hence it may be necessary to design for M_{cy} as well as for M_{cx}.

For an open grid the value of Poisson's ratio may be considered approximately zero and accordingly the value $D_1 = 0$ was used in (6). In Figure 6 a

7. AASHO Standard Specifications for Highway Bridges, 1949, Art. 3.3.5(b) and 3.3.2.

comparison of the curves obtained for $\frac{M_{cx}}{P}$ by Kuo with the 1949 AASHO specifications is shown. The coefficient $\frac{M_{cx}}{P}$ is shown plotted in solid lines against span S for $\frac{s}{\sqrt{r}} = 0$ and .1 and $r = .01, .02,$ and .04 covering the range of values of these parameters which would occur in the typical open-grid slabs. The AASHO specifications[8] yield the straight dashed lines covering the usual design range of weight and spacing b_1 between main bars. Kuo points out that the dependence of the AASHO specification formulas on the variables of weight and spacing appears to be inappropriate for the design problem since the theoretical curves for the coefficient $\frac{M_{cx}}{P}$ shows dependence only on the anisotropy parameters.

Figures 4, 5 and 6 showing the bending moment coefficient curves are taken from Kuo's paper. Curves showing the variation of the coefficient $\frac{M_{cy}}{P}$ for an open grid with k, r and $\frac{s}{\sqrt{r}}$ are also given by Kuo in a series of charts but these will not be repeated here. In the next section we will, however, include the approximation formula proposed by Kuo for $\frac{M_{cy}}{P}$ for open grids.

Proposed Formulas for Moments and Deflections

Concrete-filled Steel Grid Floors

The following formulas were proposed by Kuo as furnishing good approximations to the maximum positive values of the bending moment coefficients $\frac{M_{cx}}{P}$ and $\frac{M_{cy}}{P}$ in a one-way continuous concrete-filled steel grid slab under a wheel load P uniformly distributed over an area of $1.25' \times 1.25'$. They represent the moment coefficients at the center of the loaded panel of a two-panel slab as in Figure 2.

$$\left.\begin{aligned} \frac{M_{cx}}{P} &= \frac{S}{1.4(\log_{10} r)\cdot(S+2.5)+19}, \quad \text{for } 1.25' \leq S \leq 2.5' \\ \frac{M_{cx}}{P} &= \frac{S}{2.6\, r^{0.28}\cdot(S+2)+7.5}, \quad \text{for } 2.5' \leq S \leq 12.5' \end{aligned}\right\} \quad (19)$$

$$\left.\begin{aligned} \frac{M_{cy}}{P} &= 0.072\, r^{0.21}\cdot\sqrt{S-1.15}, \quad \text{for } 1.25' \leq S \leq 4.0' \\ \frac{M_{cy}}{P} &= \frac{S}{3r^{-0.09}\cdot S + 20\, r^{-0.30}}, \quad \text{for } 4.0' \leq S \leq 12.5' \end{aligned}\right\} \quad (20)$$

8. AASHO Standard Specifications for Highway Bridges, 1949, Art. 3.3.5.(c).

where S = span in feet, $r = \frac{D_y}{D_x}$ and $.1 \leq r \leq 1$ for concrete-filled slabs. The parameter $\frac{S}{\sqrt{r}}$ has been taken equal to 1.0.

We propose now the following formulas which furnish good approximations to the maximum value of the deflection coefficient w_c^* and the maximum negative moment - the negative edge-moment M_e^*- for a concrete-filled steel grid slab. The formula for w_c^* represents the deflection coefficient at the center of the loaded panel of a two-panel slab as in Figure 2. The formula for M_e^* represents the moment coefficient at the center of the intermediate edge in a two-panel slab, each panel subject to the central load P as in Figure 3.

$$w_c^* = (.015 - .008k) - (.011 - .009k)\cdot \log_{10} r \qquad (21)$$

$$-M_e^* = (.146 - .041\,k^{1.20}) - (.036 - .034\,k^{1.51})\cdot \log_{10} r \qquad (22)$$

where for convenience we have used the variable k rather than span S, the relation between k and S being $kS = 1.25'$ as before, when S is measured in feet. The formula for w_c^* furnishes a good approximation to Kuo's data over the range $.1 \leq r \leq 1.0$, $.1 \leq k \leq 1.0$ (or equivalently $1.25' \leq S \leq 12.5'$) the error being of the order of 2% for intermediate values of k and r and increasing to the order of 10% for the smallest spans where $k = 1$ or $S = 1.25'$. The formula for M_e^* furnishes a good approximation to Kuo's data for $.2 \leq r \leq 1.0$, $.1 \leq k \leq \frac{1}{2}$ $(2.50' \leq S \leq 12.5')$, the error being of the order of 2%. As in Kuo's formulas, the parameter ratio $\frac{S}{\sqrt{r}}$ has been assumed equal to 1.0 and so does not appear explicitly.

The variation of w_c^* with k (or S) for different values of r over the range for concrete-filled slabs is shown in the curves of Figure 7. Figure 8 shows the variation of M_e^* with k (or S) for different r values over the range for concrete-filled slabs. The data for these curves was obtained from Kuo's tabulations.[5]

Open-grid Steel Floors

The following formulas were proposed by Kuo to approximate the maximum positive values of the bending moment coefficients $\frac{M_{cx}}{P}$ and $\frac{M_{cy}}{P}$ in a one-way continuous open-grid steel slab under a wheel load P as before:

$$\left.\begin{aligned} \frac{M_{cx}}{P} &= (0.325 - 0.7\sqrt{r})\cdot(S-1) + 0.1 \quad , \quad 1.5' \leq S \leq 3.75' \\ \frac{M_{cx}}{P} &= \frac{S}{0.25\,r^{-0.29}\cdot(S+2) + 4.5} \quad , \quad 3.75' \leq S \leq 12.5' \end{aligned}\right\} \qquad (23)$$

$$\frac{M_{cy}}{P} = 0.37\, r^{0.38} \cdot S + 0.01 \quad , \quad 2.0' \leq S \leq 12.5' \qquad (24)$$

where again $S=$ span in feet and $r = \frac{D_y}{D_x}$. Now, the range of r for an open-grid floor to which these formulas apply is $.01 \leq r \leq .04$. The parameter $\frac{s}{\sqrt{r}}$ has been fixed equal to .05 as an average value for open-grid floors.

We propose now the following formulas which furnish good approximations to the maximum value of the deflection coefficient w_c^* and the maximum negative edge-moment M_e^* for an open-grid floor. As in the case of concrete-filled steel grid floors, the formula for w_c^* represents the deflection coefficient at the center of the loaded panel of a two-panel slab and M_e^* represents the moment coefficient at the center of the intermediate edge in a two-panel slab, each panel subject to the central load P. The rigidity ratio r is restricted to the range $.01 \leq r \leq .04$ and the parameter $\frac{s}{\sqrt{r}}$ has been assigned the average value .05 following Kuo.

$$\left.\begin{aligned} w_c^* &= (.011 + .028k - .027k^2) - [.019(1.20-k)^{2.8} - .002] \cdot \log_{10} r \,, \\ &\qquad \text{for } \tfrac{1}{3} \leq k \leq 1 \,, (1.25' \leq S \leq 3.75') , \\ w_c^* &= (.073k - .008) - (.037 - .078k) \cdot \log_{10} r \,, \\ &\qquad \text{for } \tfrac{1}{10} \leq k \leq \tfrac{1}{3} \,, (3.75' \leq S \leq 12.5') . \end{aligned}\right\} \qquad (25)$$

$$\left.\begin{aligned} -M_e^* &= [.30 - .44(k-.33)^{2.8}] - [.21(1-k)^{3.3} - .003] \cdot \log_{10} r \,, \\ &\qquad \text{for } \tfrac{1}{3} \leq k \leq 1 \,, (1.25' \leq S \leq 3.75') , \\ -M_e^* &= [.30 - 10.(.33-k)^{4.6}] - (.114 - .180k) \cdot \log_{10} r \,, \\ &\qquad \text{for } \tfrac{1}{10} \leq k \leq \tfrac{1}{3} \,, (3.75' \leq S \leq 12.5') . \end{aligned}\right\} \qquad (26)$$

The formulas for M_e^* furnish values which deviate from Kuo's data by less than 2%. The formulas for w_c^* furnish values which agree with Kuo's to within about 2% except near the extreme value of $k=1$ $(S = 1.25')$ where the deflection coefficient is small and the error increases to the order of 10%.

It is of course possible to approximate a given set of data by any number of mathematical formulas which differ widely in form. The type of approximation formulas given here for w_c^* and M_e^*, however, appeared quite natural for the given data for reasons as follows. Plots of w_c^* and M_e^* vs. $\log_{10} r$ for

different values of k appeared as straight lines on semi-logarithmic paper for the ranges of r and k considered. Thus, the desired approximation formulas were of the form

$$A \cdot \log_{10} r + B$$

where A and B could be read off as functions of k from the straight lines plotted. Where A and B were not sufficiently well approximated by a linear function of k, either a quadratic function or a power law, e.g. $A - A_o = (k - k_o)^p$, was used, the latter being conveniently obtained through the use of logarithmic paper.

The variation of W_c^* with k (or S) for different values of r over the range for open-grid steel floors is shown in the curves of Figure 9. Figure 10 shows the variation of M_e^* with k (or S) for different r values over the range for open-grid steel floors. The data for these curves was taken from Kuo's tabulations.

Estimate of Rigidities

In order to apply the formulas given in Section 3 to determine the coefficients $\frac{M_{cx}}{P}$, $\frac{M_{cy}}{P}$, W_c^* and M_e^* it is only necessary to substitute in the desired value of the span S in feet or the fraction $k = \frac{1.25'}{S}$ and the value of the rigidity ratio r for a given floor. The estimation of the rigidity ratio $r = \frac{D_y}{D_x}$ for practical design purposes was discussed by Kuo. For the case of <u>concrete-filled steel grid floors</u> he puts

$$D_x = \frac{E_c \cdot I_x}{b_1 \cdot (1-\mu^2)} , \qquad D_y = \frac{E_c \cdot I_y}{a_1 \cdot (1-\mu^2)} \tag{27}$$

where b_1 is the spacing between main bars of the grid (measured in the y direction) and a_1 is the spacing between the cross bars (measured in the x direction), E_c and μ are the modulus of elasticity and Poisson's ratio for concrete and I_x and I_y are the sectional moments of inertia of an elementary block of size $a_1 \times b_1$ of the grid as computed by the usual method of transformed sections used for reinforced concrete. For the case of <u>open-grid steel floors</u> Kuo puts $\mu = 0$ to obtain

$$D_x = \frac{E\, I_x}{b_1} , \qquad D_y = \frac{E\, I_y}{a_1} \tag{28}$$

where a_1 and b_1 are as defined above, E is the modulus of elasticity for the

steel and I_x and I_y are the sectional moments of inertia of an elementary block of size $a_1 \times b_1$ of the grid.

Having determined the coefficients, the actual moments and deflections can be obtained for a given load P. In the case of the bending moments this amounts to just multiplying the coefficient by the load. In the case of the deflection we have

$$W_c = \frac{PS^2}{D_x} \cdot w_c^*$$

where D_x is determined either by (27) or (28) depending on the type of floor.

CONCLUSION

The formulas (19), (20), (23), (24) which Kuo proposed for the determination of the maximum positive moments in concrete-filled grid and open grid steel bridge floors under wheel loads have been supplemented by formulas (21), (22), (25), (26) for the determination of the maximum deflection and maximum negative moment in both of these types of floors. These formulas are the first to take into account the dependence of the coefficients of moment and deflection on the degree of anisotropy of the floor as measured by the ratio $r = \frac{D_y}{D_x}$ of the flexural rigidities in the two principal directions. These formulas are based on the results of the complete theoretical analysis of the deflection of an anisotropic, one-way continuous, rectangular slab under concentrated loads carried through by Kuo and summarized in Section 2 of this paper.

The theory and the formulas given can ultimately be verified only by correlation of the predicted results with the results of a properly designed experimental program. Subject to the results of such actual tests it is proposed that these formulas be used as a basis for the design of steel grid highway bridge floors.

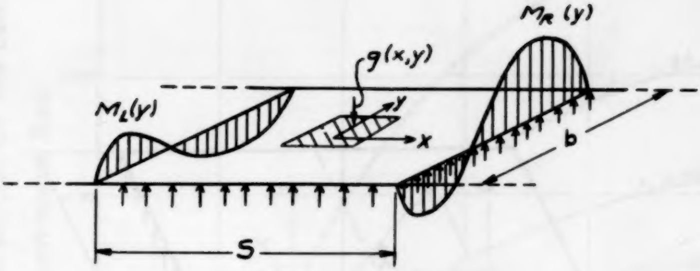


Fig. 1

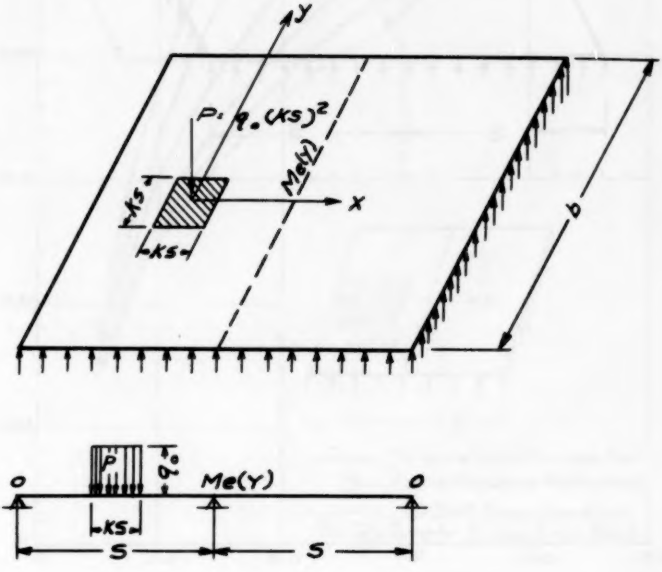


Fig. 2

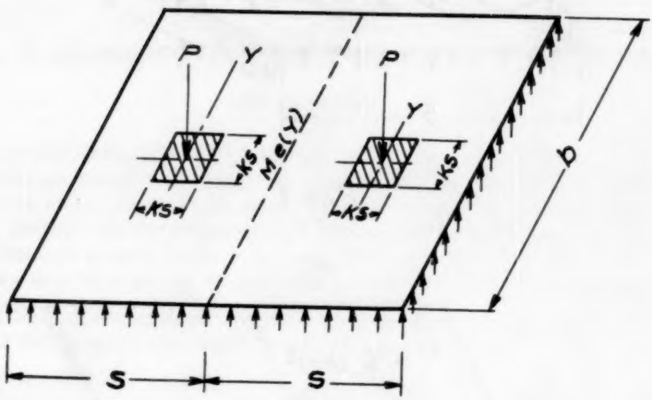
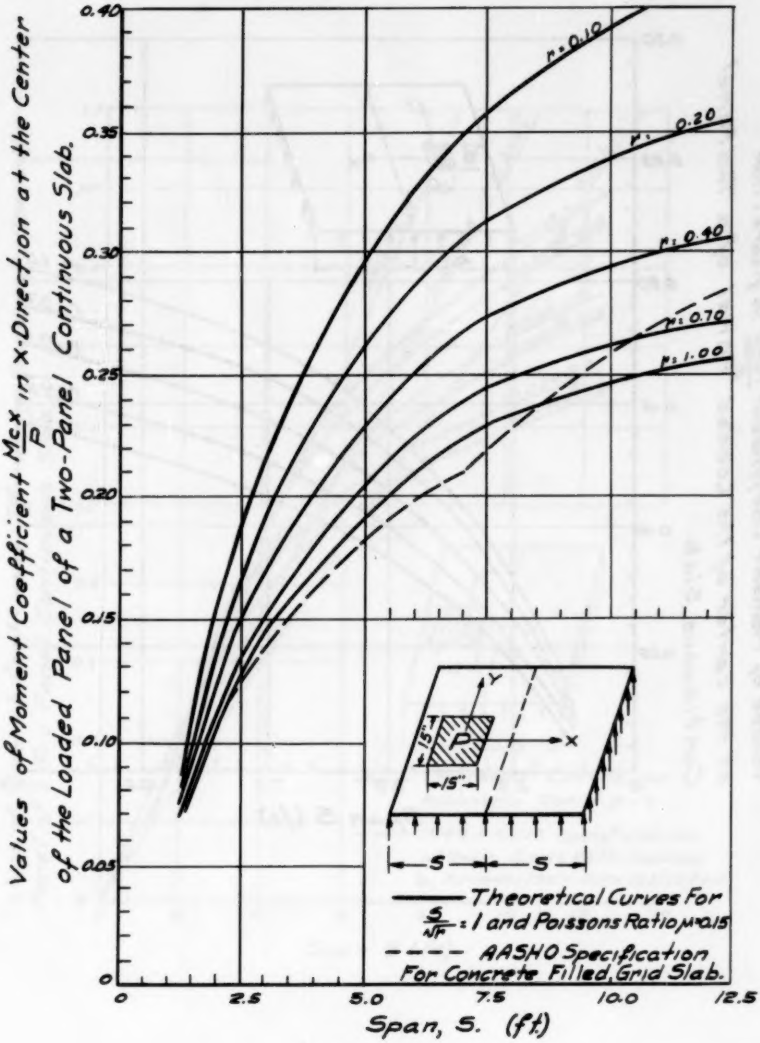
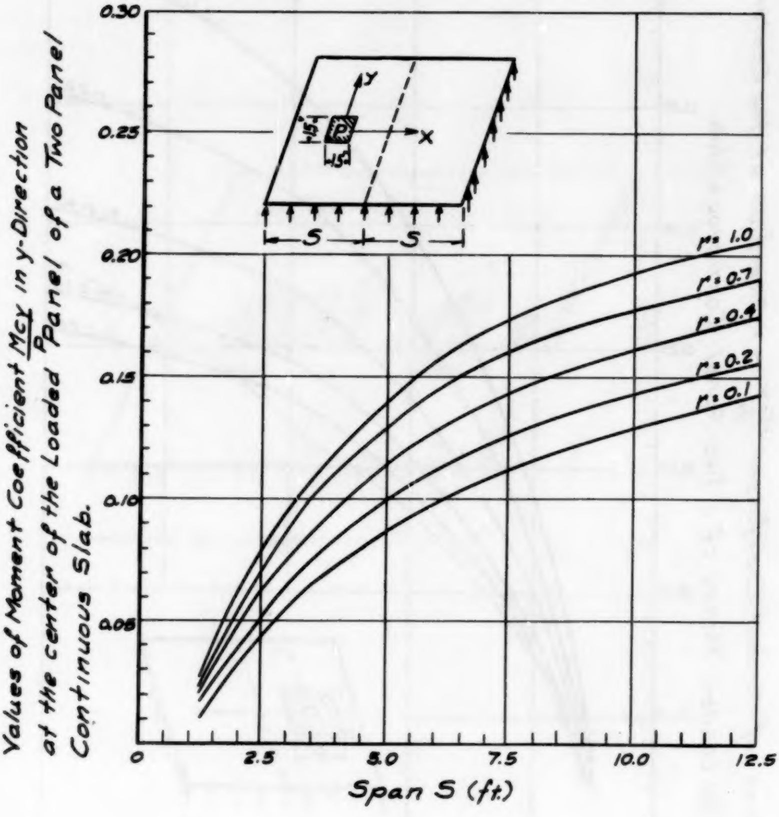


Fig. 3



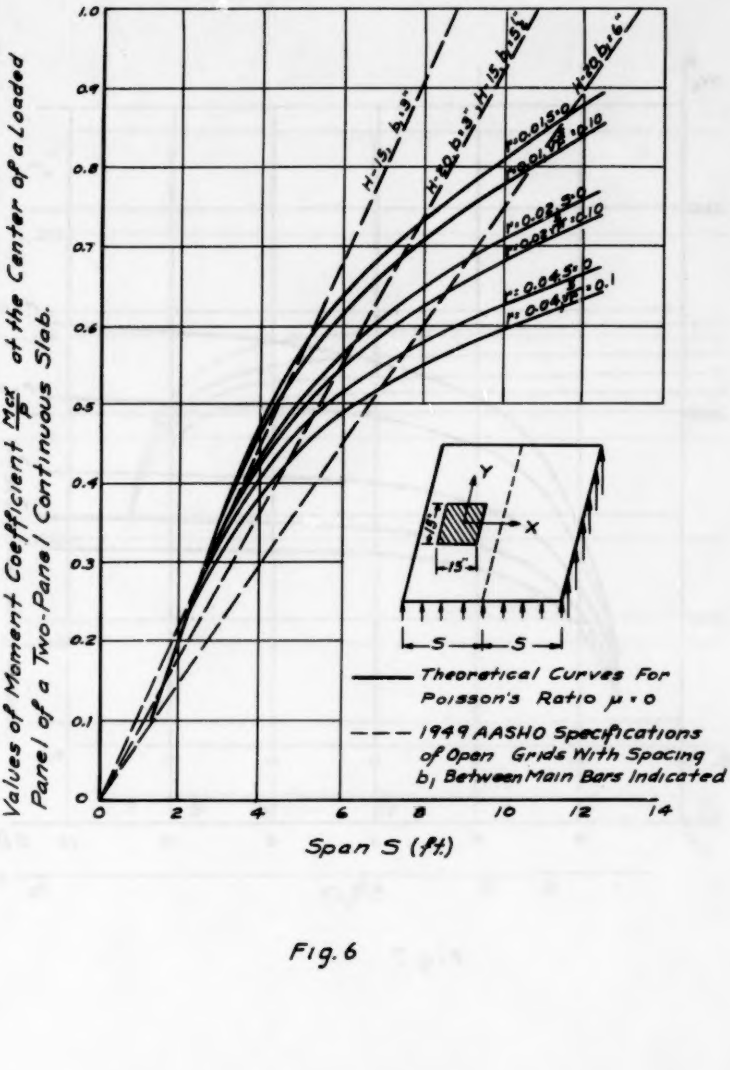
Values of Moment Coefficient $\frac{M_{cx}}{P}$ in X-Direction at the Center of the Loaded Panel of a Two-Panel Continuous Slab.
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
r = 0.10
r = 0.20
r = 0.40
r = 0.70
r = 1.00
Y
X
P
15"
15"
S
S
Theoretical Curves For $\frac{S}{\sqrt{r}} = 1$ and Poissons Ratio $\mu = 0.15$
AASHO Specification For Concrete Filled Grid Slab.
0
2.5
5.0
7.5
10.0
12.5
Span, S. (ft)

Fig. 4



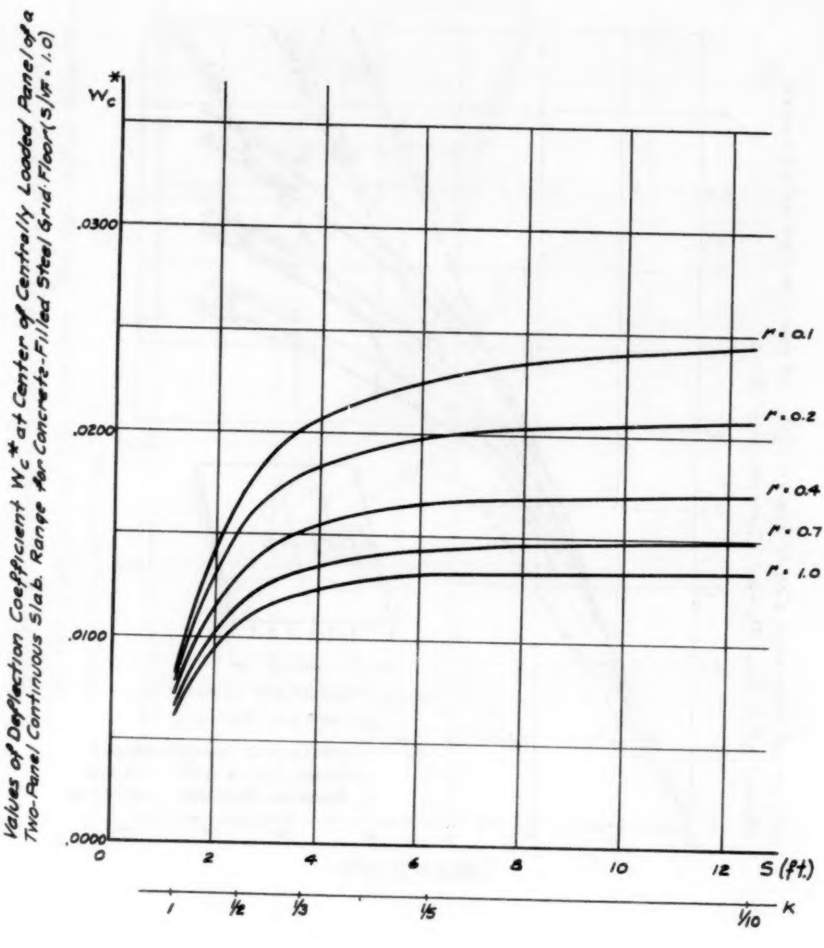
Values of Moment Coefficient $\frac{M_{cy}}{P}$ in y-Direction at the center of the Loaded Panel of a Two Panel Continuous Slab.
0.30
0.25
0.20
0.15
0.10
0.05
0
2.5
5.0
7.5
10.0
12.5
Span S (ft.)
μ = 1.0
μ = 0.7
μ = 0.4
μ = 0.2
μ = 0.1
Y
X
P
S
S

Fig. 5



Values of Moment Coefficient $\frac{M_{cx}}{P}$ at the Center of a Loaded Panel of a Two-Panel Continuous Slab.
Span S (ft.)
H-15, b_1=3"
H-20, b_1=3"
H-15, b_1=1.5"
H-20, b_1=6"
Theoretical Curves For Poisson's Ratio μ = 0
1949 AASHO Specifications of Open Grids With Spacing b_1 Between Main Bars Indicated
X
Y
15"
S

Fig. 6



Values of Deflection Coefficient Wc* at Center of Centrally Loaded Panel of a Two-Panel Continuous Slab. Range for Concrete-Filled Steel Grid Floor (S/VF = 1.0)
Wc*
.0300
.0200
.0100
.0000
0
2
4
6
8
10
12
S (ft.)
1
1/2
1/3
1/5
1/10
K
r = 0.1
r = 0.2
r = 0.4
r = 0.7
r = 1.0

Fig. 7

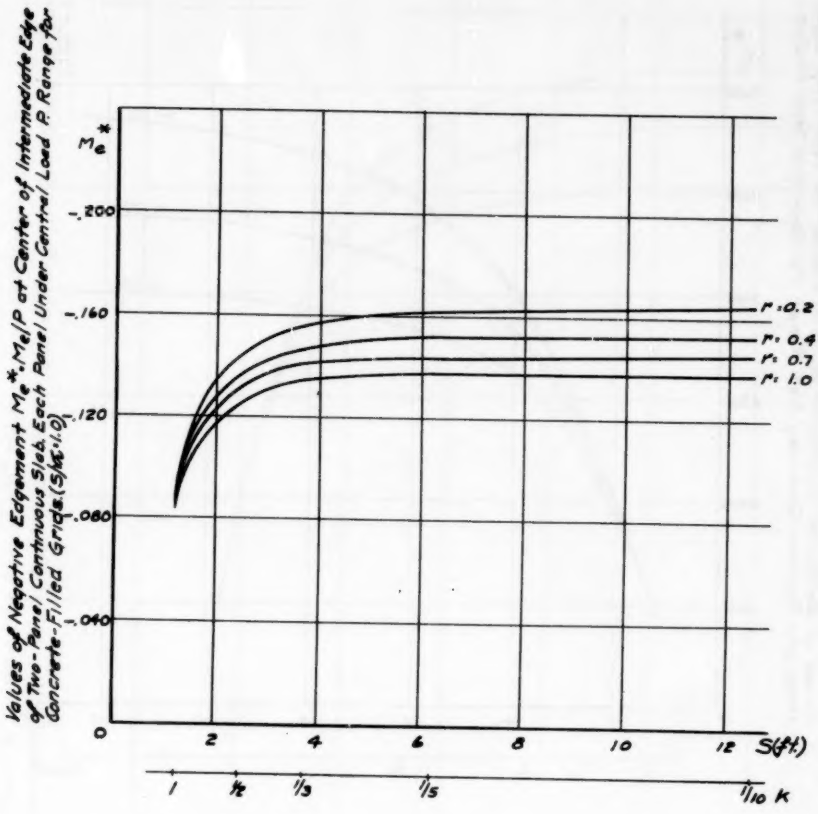
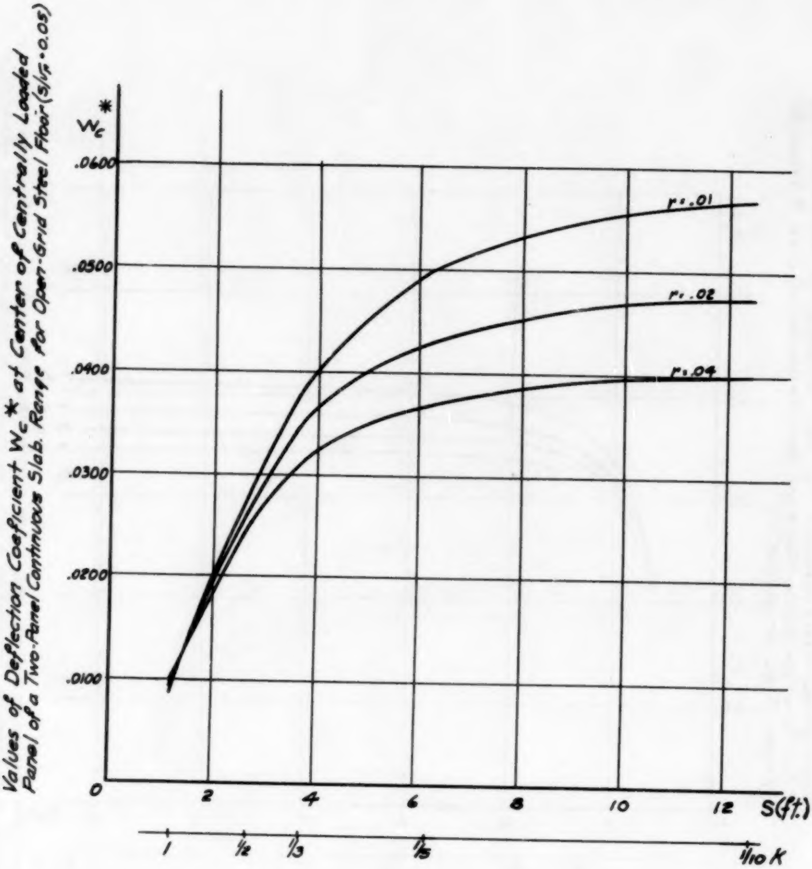

Values of Negative Edgement M_e^* = Me/P at Center of Intermediate Edge of Two-Panel Continuous Slab. Each Panel Under Central Load P. Range for Concrete-Filled Grids. (S/bc = 1.0)
M_e^*
-.200
-.160
-.120
-.080
-.040
0
2
4
6
8
10
12
S(ft.)
1
1/2
1/3
1/5
1/10
k
r = 0.2
r = 0.4
r = 0.7
r = 1.0

Fig. 8



Values of Deflection Coefficient W_c^* at Center of Centrally Loaded Panel of a Two-Panel Continuous Slab. Range for Open-Grid Steel Floor ($S/r_F = 0.05$)
W_c^*
.0600
.0500
.0400
.0300
.0200
.0100
0
2
4
6
8
10
12
S(ft.)
1
1/2
1/3
1/5
1/10 K
r = .01
r = .02
r = .04

Fig. 9

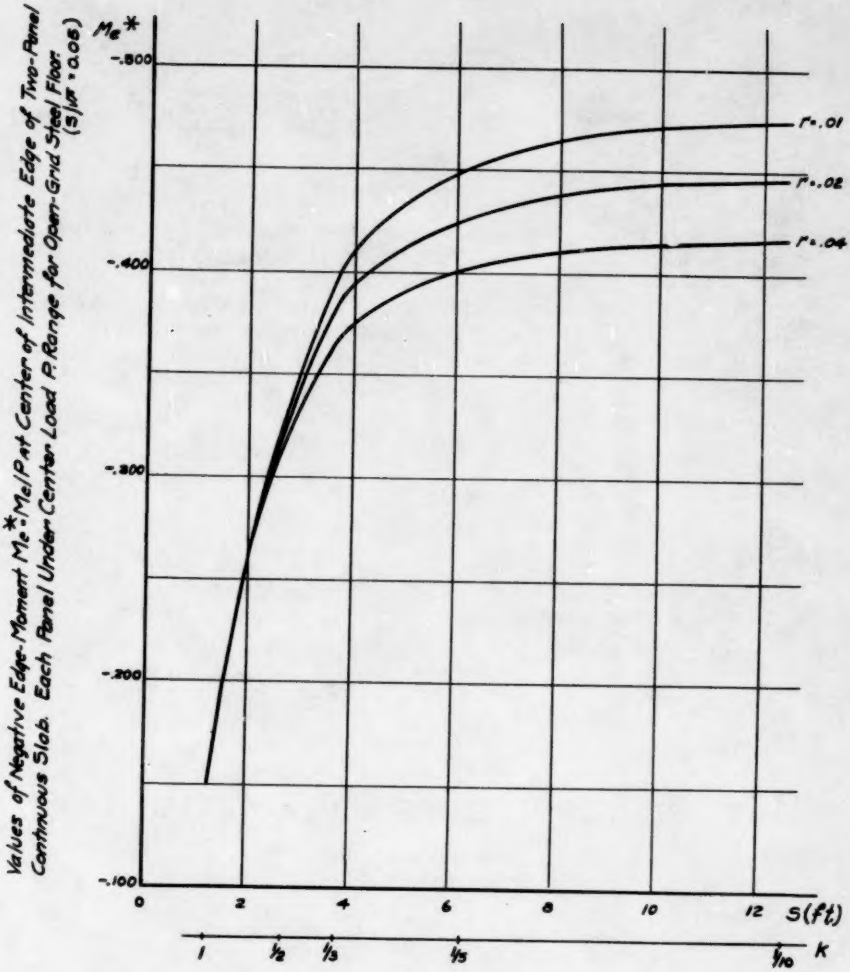

Values of Negative Edge-Moment Me* = Me/P at Center of Intermediate Edge of Two-Panel
Continuous Slab. Each Panel Under Center Load P. Range for Open-Grid Steel Floor.
Me*
-.500
-.400
-.300
-.200
-.100
0
2
4
6
8
10
12
S(ft)
1
1/2
1/3
1/5
1/10
k
r = .01
r = .02
r = .04

Fig. 10